TIME: 2 Hrs   **Max Marks: 100**

Answer any **FIVE** of the following Questions: $5 \times 20 = 100$

UNIT-I

1. Define axiomatic probability. If A and B are independent events, then show that
   i) A and $\bar{B}$ ii) $\bar{A}$ and B iii) $\bar{A}$ and $\bar{B}$ are also independent

2. The probabilities of X, Y and Z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively.

   (i)   What is the probability that the bonus scheme will be introduced?

   (ii)   If the bonus scheme has been introduced, what is the probability that the manager appointed was X?

UNIT-II

3. Define random variable and state its properties. Also explain about types of random variables along with their probability functions.

4. Define distribution function of a random variable. A continuous random variable X has a probability density function $f(x) = 3x^2; 0 \leq x \leq 1$. Find 'a' and 'b' such that
   (i) $P(X \leq a) = P(X > a)$ and
   (ii) $P(X > b) = 0.05$

UNIT-III

5. Define mathematical expectation of a random variable. Also explain any five properties of expectation.

6. Explain Characteristic function of a random variable X along with its properties

UNIT-IV

7. The joint probability density function of X and Y is given by
   $$f(x,y) = Ae^{-x-y}; 0 \leq x \leq y \text{ and } 0 \leq y < \infty.$$
        i.   Determine A
        ii. Show that the total probability is equal to unity
        iii. Find marginals.
        iv. Find conditional probability density function of Y given X=2.
        v.   Check the independency.

8. Define conditional probability function. Also Show that the covariance is independent of change of origin but not scale

UNIT-V

9. State and prove Chebychev's inequality. A random variable X   takes the values $-1,1,3,5$ with associated probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2}$. Find an upper bound to the probability $P(|X-3| \geq 1)$ by applying chebyshev's inequality

10. Define the following
    (i) Convergence in Probability   (ii)   Weak Law of Large Numbers
    (iii) Bernoulli's Law of Large Numbers   (iv)   Central Limit Theorem